General Relativity , Energy , Mass and Infinitely Extended Particles

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1.Introduction

As we know , theories of elementary particles for calculating particles' masses are less successful .

Many people have attempted to achieve a way to calculate particles' masses [1] ,[2] . But none of them could present an analytic way for all states of particles . They merely could present limited ways , without a basic expression. Mahmoud Hessaby [3] , the professor of Tehran university , when he studied in Paris university (1923 – 1927) , began to study about sensitivity of photoelectric cells and elementary particles . His supervisor was professor Fabry [4] while defending his PhD thesis , and one of his classmates was famous scientist , " Louis De Broglie " [5].

Dr.Hessaby studied De broglies's theory (wave-particle property of matter) and general relativity for the first time in 1925 .

He continued his studies with Albert Einstein's supervision , in Princeton university and with Enrico Fermi [6] , in Chicago , from 1946 to 1949 .

He focused on elementary particles , general relativity , fields theory and energy density .

Later in 1977 , he presented his study results on calculation of energy density and particles' masses .

2. Basic considerations

In general relativity , curvature of space , and line elements are as below :

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (1)$$

where $G_{\mu\nu}$ is curvature of space tensor , G is Newton's constant , c is light velocity and $T_{\mu\nu}$ is stress-energy tensor.

In a flat spacetime , ordinary velocity of particles are ;

$$u = \frac{dx}{dt}e_x + \frac{dy}{dt}e_y + \frac{dz}{dt}e_z \qquad (2)$$

$\{ e_x , e_y , e_z \}$ are basis vectors , as well as orthonormal basis .

According to the relativistic description , particles exist in a four dimensional spacetime , the basis vectors would be $\{ e_t , e_x , e_y , e_z \}$.

In this description , the ordinary velocity of particles is not a vector.

Instead one defines a four-velocity ;

$$u = -c\frac{dt}{d\tau} e_t + \frac{dx}{d\tau} e_x + \frac{dy}{d\tau} e_y + \frac{dz}{dt} e_z \qquad (3)$$

Where τ is the proper time of the particle, i.e. the time is measured by a standard clock carried by the particle using Einstein's summation convention, we may write ;

$$U = U^{\mu}e_{\mu} = \frac{dx^3}{d\tau}e_{\mu} \quad , x^{\mu} \in \{ x^0 , x^1 , x^2 , x^3 \}$$

Where $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = y$.

If $\frac{dt}{d\tau} = \gamma$, then ;

$$U = \gamma (c , u^x , u^y , u^z) \qquad (4)$$

$$U = \gamma (c , u) \qquad (5)$$

In the next frame of the particle , u=o and γ =1 .

Hence the four-velocity reduces to :

$$U = ce_t \qquad (6)$$

In this frame , the particle moves in the time direction with the velocity of light.

The four-momentum , P , of a particle with rest-mass m_o is defined by ;

$$P = m_o U \qquad (7)$$

The ordinary three-dimensional relativistic momentum of the particle is ;

$$P=mu=\gamma m_o u \qquad (8)$$

From Eqs.(6) and (8) , follows ;

$$P=(E/C\ ,p\ ) \qquad (9)$$

Where the total energy of the particle is , E.

The four-force , or the Minkowski [7] force F is defined by ;

$$F=\frac{dP}{d\tau} \qquad (10)$$

The ordinary force f is ;

$$f=\frac{dP}{dt} \qquad (11)$$

It follows that ;

$$F = \gamma\ (\frac{dE}{cdt}, \frac{dP}{dt}) = \gamma\ (\frac{f.u}{c}, f\ ) \qquad (12)$$

In the rest frame of the particle ;

$$F_0 = (\ 0\ ,\ f_0\ ) \qquad (13)$$

Where f_0 is the Newtonian force on the particle , The four-acceleration "A"of the particle is ;

$$A=\frac{dU}{d\tau} \qquad (14)$$

In the case that the rest mass is constant we get ;

$$A = \frac{1}{m_o} F \qquad (15)$$

The ordinary acceleration "a" is ;

$$a = \frac{du}{dt} \qquad (16)$$

Using that ;

$$F = m_o \frac{d}{dt}(\gamma\ u) = \gamma\ m_o (\ a + \gamma^2 \frac{u.a}{c^2} u) \qquad (17)$$

We find from Eqs.(12) , (14) and (17) ;

$$A = \gamma^2 (\gamma^2 \frac{u.a}{c}, a + \gamma^2 \frac{u.a}{c^2} u) \qquad (18)$$

The component expressions (12) and (18) are valid only with respect to an orthonormal basis field .

Curved space one can always introduces local Cartesian coordinate systems with orthonormal basis coordinate vector fields .

Line-element of Minkowski [7] spacetime in spherical coordinates has the form (in units with c=1) ;

$$ds^2 = -dt^2 + d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (19)$$

To solve the field equation for empty spacetime with static and spherically symmetric 3-space is reasonable to assume that the line-element can be written ;

$$ds^2 = -f(\bar{r})dt^2 + g(\bar{r})d\bar{r}^2 + h(\bar{r})\bar{r}^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (20)$$

With a new radial coordinate $r = \bar{r}\sqrt{h(\bar{r})}$, line-element becomes ;

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (21)$$

To replace the functions A(r) and B(r) by exponential functions in order to obtain somewhat simpler expressions for the components of the Einstein's tensor , we can use the following :

$$e^{2a(r)} = A(r) \text{ and } e^{2\beta(r)} = B(r)$$

We find ;

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (22)$$

These coordinates are called Schwarzschild coordinates .

Professor Hessaby assumed the line-element as below ;

$$ds^2 = -e^{\delta_1}dt^2 - e^{a_1}dr^2 r^2(e^{\beta_1}d\theta^2 + e^{\gamma_1}\sin^2\theta d\phi^2) \quad (23)$$

After to compare eqs.(22) and (23) we will find ;

$$e^{\delta_1} = -e^{2\alpha_1}, e^{\alpha_1} = -e^{2\beta_1}, e^{\beta_1} = -1, e^{\gamma_1} = -1$$

In general relativity (GR) , the relationship between matter and the curvature of spacetime is contained in equation , which is Einstein's equation .

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G T_{\mu\nu}}{c^4} \qquad (24)$$

Where R $\mu\nu$ is Ricci curvature tensor , R is scalar curvature , g $\mu\nu$ is metric tensor, G is Newtone's constant , c is light velocity and $T_{\mu\nu}$ is stress-energy tensor .

We could use eqs.(24) as below ;

$$T_{\mu\nu} = \frac{c^4}{8\pi G} (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) \qquad (25)$$

And we write ;

$$a = \frac{+c^4}{2G} \qquad (26)$$

where a is a dimensional constant .

And then we find ;

$$T_{\mu\nu} = \frac{a}{4\pi} (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) \qquad (27)$$

The contracted energy tensor is :

$$T = U = \sum u_{\mu\nu} = \sum T_{\mu\nu} \qquad (28)$$

Since we have identically U=0 , the identification of T_μ^ν with U_μ^ν , gives by contraction ;

$$U = T = \frac{a}{4\pi} R = 0 \qquad (29)$$

So that we have identically R = 0 .

Taking account of (29) , relation (27) becomes ,

$$U_{\mu\nu} = T_{\mu\nu} = \frac{a}{4\pi} R_{\mu\nu} \qquad (30)$$

With refer to Professor Hessaby's paper , $U_{\mu\nu}$ would be ,

$$U_{11} = - U_{22} = - U_{33} = U_{44} = \frac{1}{4\pi} E^{-(\alpha_1+\delta_1)} \times (F_{14})^2 \qquad (31)$$

Where $F_{\mu\nu}$ is generalized field on gravitational , electric , and nuclear fields .

Relations (30) and (31) give ;

$$R_{11} = - R_{12} = - R_{23} = R_{44} = \frac{1}{a} e^{-(\alpha_1+\delta_1)} (F_{14})^2 \qquad (32)$$

$R_{\mu\nu}$ is in terms of $\alpha_1, \beta_1, \gamma_1, \delta_1$.

The components $T_{\mu\nu}$ of the energy-momentum tensor , or sometimes the stress-energy tensor , are " the flux of the μ^{th} component of momentum in the $\nu^{\mu h}$ direction . "

When we are in vaccum- no energy or momentum , $T_{\mu\nu}$ = 0 and becomes Einstein's equation in vaccum ;

$$R_{\mu\nu} = o \qquad (33)$$

This is somewhat easier to solve than the full equation . Although the full nonlinear Einstein's equation looks simple , in application it is not . If we recall the definition of the Riemann [8] tensor in terms of the Christoffel symbols , and the definition of those in terms of the metric , we will realize that Einstein's equation for the metric are complicated indeed.

Through christoffel symbols (Non-vanishing symbols) and by relations (29) the contracted tensor R is zero ;

$$R = \sum R_{\mu\nu} \qquad (34)$$

Based on $R_{22} = R_{33}$, we assume the equality of β_1 and γ_1 (Refer to Professor Hessaby's paper)

Making use of the equality $R_{11} = R_{44}$, we obtain ;

$$e_1^{-\alpha} (\beta_1'' + \frac{1}{2} \beta_1'^2 + 2 \frac{\beta_1'}{r} - \frac{1}{2} \alpha_1' \beta_1 - \frac{1}{2} \beta_1' \delta_1' - \frac{\alpha_1'}{r} - \frac{\delta_1'}{r}) = o \qquad (35)$$

in the first case where $\beta_1 = \gamma_1 =$ ٠ (Professor Hessaby's paper) and relation (35) gives then $\delta_1 = - \alpha_1$, becomes ;

$$e^{\delta_1} (-\delta_1'' - \delta_1'^2 - 4\frac{\delta_1'}{r} - \frac{2}{r^2}) + \frac{2}{r^2} = 0 \qquad (36)$$

the solution

$$e^{\delta} = (1 + \frac{k}{r})^2 \qquad (37)$$

where k is a constant depending on mass of the particle and having the dimensions of a length , then we have ;

$$e^{\alpha} = e^{-\delta} = (1 + \frac{k}{r})^{-2} \qquad (38)$$

We have assumed $\beta_1 = \gamma_1 = 0$, the metric becomes ,

$$ds^2 = - \frac{1}{(1+\frac{k}{r})^2} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\theta^2 + (1 + \frac{k}{r})^2 dt^2 \qquad (39)$$

In the second case , if we extend the validity of the equality $\delta_1 = -\alpha_1$ to the general case where $\beta_1 \neq 0$, the differential equation (35) reduces to ;

$$\beta_1'' + \frac{1}{2}\beta_1'^2 + 2\frac{\beta_1'}{r} = 0 \qquad (40)$$

After solving the equation above the metric conclusion , changes into :

$$ds^2 = - (1 + \frac{A}{r})^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\theta^2) + \frac{1}{(1+\frac{A}{r})^2} dt^2 \qquad (41)$$

3.The Fields

For gravitational field , we consider the form of the metric as found in (39) . The four potential have known by ϕ_μ . The field in general case is given by the expression ;

$$F_{\mu\nu} = \frac{\partial \phi_\mu}{\partial x_\nu} - \frac{\partial \phi_\nu}{\partial x_\mu} \qquad (42)$$

The field F_{14} is given by equ (31) , which $\delta_1 = -\alpha_1$

$$U_{44} = \frac{1}{2\pi} e^{-(\alpha+\delta)} (F_{14})^2 = \frac{1}{4\pi} (F_{14})^2 \qquad (43)$$

Also the energy density would be ;

$$U_{44} = -\frac{a}{4\pi} R^{44} = -\frac{a}{4\pi} g^{44} R_4^4 = -\frac{a}{4\pi} e^{-\delta_1} R_4^4 \qquad (44)$$

A result of equ .(44) , which $\delta_1 = -\alpha_1$

And $\beta_1 = \gamma_1 = 0$ is ;

$$R_4^4 = e_1^{\delta} \left(-\frac{1}{2}\delta_1'' - \frac{1}{2}\delta_1' - \frac{\delta_1'}{r}\right) \qquad (45)$$

If $e^{\delta_1} = \left(1 + \frac{k}{r}\right)^2$, so that we have ;

$$R_4^4 = -\frac{k}{r^4} \qquad (46)$$

And then ;

$$F_{14} = \frac{\sqrt{a}k}{r^2} \qquad (47)$$

The energy density , becomes ;

$$U^{44} = \frac{a}{4\pi} \frac{k^2}{r^2 (r+k)^2}$$

And the integral of energy density over the whole space is ;

$$W = \frac{a}{4a} k^2 \int_0^\infty \frac{1}{r^2 (r+k)^2} r^2 \sin^2 \theta d\theta d\phi dr = ak \qquad (48)$$

The mass of the particle being " m " , must be equal to ;

$$ak = mc^2 \qquad (49)$$

The dimension of " K " being that of a length , and for " a " must be a force .

We set ;

$$a=\frac{c^4}{G}$$

(50)

$$k=\frac{Gm}{c^2}$$

We see that in the integral of energy density , we encounter no infinities , and the mass of the particle consists of the integral of its energy density overall space !

The metric (39) becomes now ;

$$ds^2 = -\frac{1}{(1+\frac{Gm}{c^2 r})^2}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2 + (1+\frac{Gm}{c^2 r})^2 dt^2 \qquad (51)$$

Einstein's Solution is ;

$$ds^2 = -\frac{1}{(1-\frac{2m}{r})}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2 + (1-\frac{2m}{r})dt^2 \qquad (52)$$

In the metric (51) , the $g_{\mu\nu}$ are perfect squares , and also do not have any singularities apart from the origin , giving with a finite value for the energy of the particle !

In the electric field we consider the metric (41) , and for the energy density is ;

$$u^{44} = -\frac{b}{4\pi}R^{44} = \frac{1}{4\pi}e^{-(\alpha+\delta)}(F_{14})^2 \qquad (53)$$

Since we have $\alpha_1 = \beta_1 = \gamma_1 = -\delta_1$, we get ;

$$R^{44} = e_1^{\delta_1}(-\frac{1}{2}\delta_1'' - \frac{\delta_1'}{r}) \qquad (54)$$

Replacing e^{δ_1} by $\left(1+\frac{A}{r}\right)^{-2}$ we find ;

$$R^{44} = -\frac{A^2}{(r+A)^4} \qquad (55)$$

So that

$$R^{44} = -\frac{1}{b}(F_{14})^2 = -\frac{A^2}{(r+A)^4} \qquad (56)$$

And ,

$$u^{44} = -\frac{bA^2}{4\pi(r+A)^4} \qquad (57)$$

The integral over all space is ;

$$\int u^{44} d\nu = \frac{bA}{3} \qquad (58)$$

We know ,

$$\frac{bA}{3} = mc^2 \qquad (59)$$

"A" has the dimension of length , and " b " must have the dimension of a force . We remark that the $\frac{e^2}{mc^2}$ has dimension of a length , and the combination $\frac{m^2c^4}{e^2}$ has the dimension of a force . we set ,

$$b = \frac{9m^2c^4}{e^2}$$

$$(60)$$

$$A = \frac{1}{3}\frac{e^2}{mc^2}$$

The " A " is equal to ;

$$A = \frac{e^2}{3me^2} = \frac{1}{3}\frac{e^2}{\hbar c}\frac{\hbar^0}{mc} = \frac{1}{3}\alpha\lambda = 0.9393\times10^{-3} m \qquad (61)$$

References :

[1] **Karl Otto Greulich** , " Calculation of the Masses of All Fundamental Elementary particles with an Accuracy of Approx.1% , " Journal of Modern physics , 2010 , 300-302 .

[2] **Alexander G.Kyriakos** , " on calculation of elementary particle's masses",Saint-Petersburg State institute of Technology , Russia , e-mail : agkyriak@yahoo.com .

[3] **Mahmoud Hessaby** (1903-1992) was a prominent Iranian scientist , researcher and distinguished professor of university of Tehran (refer to www.hessaby.com) .

[4] **Charles Fabry** (1867-1945) was born in Marseille , France . When he was 18, he entered the Ecole Polytechnique in Paris and , after graduating two years later , returned to his native Marseille for his Agrégé de physique (1889) . After his agrégation , which gave him the license to teach at any State secondary school , Fabry taught at Lycées in Pau , Nevers , Bordeaux , and Marseille , and finally at the Lycées Saint Louis in Paris . During this time he was preparing his doctoral dissertation on the theory of multibeam interference phenomena , a topic that had been treated as early as 1831 by George Biddell Airy (1801 – 1892) , but not with the depth and sophistication Fabry brought to the subject. In the years 1890-1892 Fabry published two papers on the visibility and orientation of interference fringes , the first of which was a joint paper with his mentor , Professor Jules Macé de Lepinay , and the second , which was accepted by the Faculty of Science at the University of Paris for his *Docteur és sciences* degree. These papers and the large number that soon followed gradually established Fabry as an authority in the field of optics and spectroscopy .

In 1894 Fabry replaced Alfred Pérot (1863-1925) as Maitre de Conférences (Lecturer) at the University of Marseille , where he spent the next 26 years , starting as an assistant in de Lepinay's laboratory . In 1904 , when de Lepinary retired , Fabry was appointed to fill his post as Professor of Physics at Marseille.

[5] **Louis De Broglie** (1892-1987) was a French physicist who made groundbreaking contributions to quantum theory . In his 1924 PhD thesis , he postulated the way nature of

electrons and suggested that all matters has wave properties . He won Nobel prize for physics in 1929 .

[6] **Enrico Fermi** (1901-1954) was an Italian theoretical and experimental physicist , best known for his work on the development of Chicago pile-1,the first nuclear reactor .

[7] **Hermann Minkowsky** (1864-1909) was a German mathematician he created and developed the geometry of number and used geometrical methods to solve problems in number theory , mathematical physics and the theory of relativity.

[8] **George Fredrich Bernhard Riemann** (1826-1866) was a German mathematician who made lasting contributions to analysis , number theory and some of them enabling the later development of general relativity .